

# The Coupling Effects of Kinematics and Flexibility on the Lagrangian Dynamic Formulation of Open Chain Deformable Links

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## Abstract

In this paper a nonlinear Lagrangian formulation for the spatial kinematic and dynamic analysis of open chain deformable links consisting of cylindrical joints that connects pair of flexible links is developed. The special cases of revolute or prismatic joint can also be obtained from the kinematic equations. The kinematic equations are described using  $4 \times 4$  matrix method. The configuration of each deformable link in the open loop kinematic chain is identified using a coupled set of relative joint variables, constant geometric parameters, and elastic coordinates. The elastic coordinates define the link deformation with respect to a selected joint coordinate system that is consistent with the kinematic constraints on the boundary of the deformable link. These coordinates can be introduced using approximation techniques such as Rayleigh-Ritz method, finite element technique or any other desired approach. The large relative motion between neighboring links are defined by a set of joint coordinates which describes the large relative translational and rotational motion between two neighboring joint coordinate systems. The origin of these coordinate systems are rigidly attached to the neighboring links at the joint definition points along the axis of motion. The geometry of the deformable links are included in the formulation by two constant parameters which accounts for the length and twist of the deformable link in its undeformed state. The kinematic equations that define the global position and velocity of an arbitrary point on a deformable link is developed in terms of the relative joint variable, constant geometric parameters, and elastic coordinates of deformable links. These kinematic equations are then used to develop the energy expression of the deformable link. The nonlinear terms that represent the dynamic coupling between the large relative motion and the small elastic deformations is identified and presented in terms of a set of time-invariant quantities that depend on the assumed displacement field and provide a systematic approach to study the spatial dynamics of open loop kinematic chains. The system differential equations are then developed and expressed in terms of these set of invariant quantities using Lagrange's equation of motion.